in nonlinear analysis, to unfold the potentially rich dynamics associated with double flutter, and to explore opportunities to utilize the instability for beneficial purposes.

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Numerical Computations of Purely Radial Flow Within Two Concentric Disks

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Introduction

THE radial flow within the gap formed by placing two flat disks together is pertinent to a number of engineering applications, such as radial diffusers, air-bearing, disk-type heat exchangers, pneumatic micrometers, and several others. Two distinct types of flow can evolve within such domains. The accelerating (or sink) flow is produced by admitting the fluid through the periphery and draining it from a centrally located outlet. This type of fluid motion is characterized by a monotonically decreasing pressure gradient. Because of the stabilizing effects of acceleration, it is also known to remain laminar even at very high Reynolds numbers or to laminarize in the case where the entering fluid is initially turbulent.[1] Decelerating (or source) flow emerges by introducing the fluid into the domain through a centrally located inlet. Mochizuki and Yang[2] reported on three distinct manifestations of such fluid motion. At low inlet Reynolds numbers, they observed a steady, laminar, unidirectional flow throughout the gap. For intermediate inlet Reynolds numbers, a decaying self controlled oscillatory flow appeared. Finally, for high inlet Reynolds numbers, a self-sustained fluctuating flow, evolving into a laminar–turbulent transition, followed by a reverse flow transformation to laminarlike conditions at larger radii was evident. In many respects, the disk flows are similar to the classical Jeffery–Hammel flows between two inclined planes.

The present Note reports on the numerical solution of purely radial converging and diverging flows between two stationary disks. Assuming a unidirectional flow, continuity and momentum equations are reduced into a third-order ordinary differential equation (ODE) involving only the radial velocity. The solution of the latter nonlinear equation is accomplished using a variation of the shooting method, developed by Keller in Ref. 3, where the governing equation is written as a system of three nonlinear first order ODEs, and the resulting system is solved as an initial value problem via the fourth-order Runge–Kutta method. The same method has been used previously by the authors to study the flow development in spherical and conical passages.[4,5] This numerical method is simple and fast compared with the conventional use of computational fluid dynamics (CFD) in solving fluid flow problems. The results obtained are compared with some available experimental data for the velocity and pressure in accelerating and decelerating flows.

Problem Formulation and Solution Technique

Consider the steady, laminar, incompressible, nonswirling, purely radial flow developed in the gap between a pair of parallel disks that is shown schematically in Fig. 1. From Ref. 6, the equations describing this type of fluid motion are as follows.

Continuity:

$$\int_0^1 g(\zeta)\,\mathrm{d}\zeta = \pm 1 \tag{1}$$

Radial momentum:

$$g''(\zeta) + \lambda^2 g^2(\zeta) = \lambda^2 \Delta\Pi \tag{2}$$

where g'' indicates the second derivative of g with respect to ζ,

$$\lambda^2 = \frac{(r^2-1)Re_r}{2r^2 \ln(r)}, \qquad \Delta\Pi = \frac{2r^2\Delta P}{r^2-1}$$

$V_r r = g(\zeta)$, where $r = r^*/R_0^*$, $\zeta = z^*/h^*$, $V_r = V_r^*/V_0^*$, V_r^* is the radial velocity component, and V_0^* is the average velocity at R_0^*, and $Re = \rho^* V_0^* h^*/\mu^*$, $Re_r = Re\,\xi$, $\xi = h^*/R_0^*$, $\Delta P = (P^* - P_0^*)/\rho^* V_0^{*2}$, where p^* is the static pressure at r^*, P_0^* is the static pressure at R_0^*, and ρ and μ are the fluid density and viscosity, respectively. The asterisks indicate a dimensional quantity, the plus and minus signs in the continuity equation represent outflow and inflow, respectively, and the rest of the parameters used are defined in Fig. 1.

On differentiation of Eq. (1) with respect to ζ,

$$g'''(\zeta) + \lambda^2 g(\zeta) g'(\zeta) = 0 \tag{3}$$

The required boundary conditions are

$$\zeta = 0 \rightarrow \frac{\mathrm{d}g(\zeta)}{\mathrm{d}\zeta} = 0 \tag{4a}$$

$$\zeta = 1 \rightarrow g(\zeta) = 0 \tag{4b}$$

The two conditions given correspond to the nonslip condition of the velocity at the plates and the symmetrical nature of the velocity

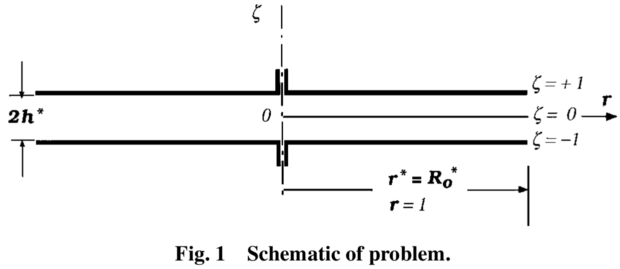

Fig. 1 Schematic of problem.

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at the midgap point. To solve the problem numerically, continuity and momentum were reduced into one differential equation for g, namely, Eq. (3), with the boundary conditions (4a) and (4b) and the integral constraint given by Eq. (1).

This nonlinear third-order ODE is written as a system of three first-order ODEs:

$$\frac{\partial \boldsymbol{G}\left(\zeta, x_1^i, x_2^i\right)}{\partial \zeta} = \boldsymbol{F}$$

where

$$\boldsymbol{G} = \begin{bmatrix} g\left(\zeta, x_1^i, x_2^i\right) \\ u\left(\zeta, x_1^i, x_2^i\right) \\ v\left(\zeta, x_1^i, x_2^i\right) \end{bmatrix}, \qquad \boldsymbol{F} = \begin{bmatrix} u \\ v \\ -2\lambda^2 g u \end{bmatrix} \tag{5}$$

with initial conditions $\boldsymbol{G} = [x_1^i \;\; 0 \;\; x_2^i]^T$.

Given x_1^i and x_2^i these equations are integrated from the centerplane $\zeta = 0$ to the upper disk $\zeta = 1$, using the fourth-order Runge–Kutta method. Because there is only one condition to satisfy at the initial point $\zeta = 0$, the shooting method, developed by Keller,[3] is employed so that x_1^i and x_2^i are the values of x_1 and x_2 at the ith iteration. Newton's method is then used to adjust the solution parameters x_1^i and x_2^i such that, at convergence, these two parameters will satisfy the conditions given in Eqs. (1) and (4b). The solution of this initial value problem will be a function of x_1^i and x_2^i, that is, $G = G(\zeta, x_1^i, x_2^i)$. Hence, we seek x_1^i and x_2^i such that, at convergence,

$$\boldsymbol{C} = \begin{bmatrix} C_1\left(x_1^i, x_2^i\right) \\ C_2\left(x_1^i, x_2^i\right) \end{bmatrix} = \begin{bmatrix} g\left(\zeta = 1, x_1^i, x_2^i\right) \\ \displaystyle\int_0^1 g\left(\zeta, x_1^i, x_2^i\right) \mathrm{d}\zeta \pm 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

and $\pm$ corresponds to inflow/outflow. Newton's method[7] is then used to obtain a better guess for $\boldsymbol{x}^i$ as follows:

$$[J]\Delta \boldsymbol{x} = -\boldsymbol{C} \qquad \text{where} \qquad [J] = \left(\frac{\partial C_i}{\partial x_j}\right)_{\text{at } \zeta = 1} \tag{7}$$

so that $\Delta \boldsymbol{x} = -[J]^{-1}\boldsymbol{C}$

$$i = 1, 2, \qquad j = 1, 2, \qquad \Delta \boldsymbol{x}^i = \boldsymbol{x}^{i+1} - x^i$$

The terms involved in the Jacobian matrix that contains $\partial g / \partial x_i$ are obtained by differentiating Eqs. (5) with respect to x_j, where $j = 1, 2$, to give

$$\frac{\partial \boldsymbol{G}_j\left(\zeta, x_1^i, x_2^i\right)}{\partial \zeta} = \boldsymbol{F}_j$$

where

$$\boldsymbol{G}_j = \frac{\partial \boldsymbol{G}}{\partial x_j}, \qquad \boldsymbol{F}_j = \begin{bmatrix} u_j \\ v_j \\ -2\lambda^2(g_j u + g u_j) \end{bmatrix} \tag{8}$$

with initial conditions

$$\boldsymbol{G}_j = [100]^T \quad \text{for } j = 1$$

$$\boldsymbol{G}_j = [001]^T \quad \text{for } j = 2$$

Once the velocity field is calculated (to be described), the pressure difference between the reference section and the current one is obtained by integrating Eq. (3) to give

$$g'' + \lambda^2 g^2 = \lambda^2 \Delta\Pi$$

where

$$\Delta\Pi = (2r^2 \Delta P)/(r^2 - 1), \qquad \Delta P = (P_0 - P)\big/\rho V_0^2$$

Given that, at $\zeta = 1$, $g = 0$, and $g'' = v(\zeta = 1)$; hence,

$$\Delta\Pi = v(\zeta = 1)/\lambda^2$$

For a given λ value, the solution proceeds as follows.

1) Assuming an initial guess $\boldsymbol{x}^0$, solve simultaneously the sets of first-order differential equations for $\boldsymbol{G}$, $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ using the fourth-order Runge–Kutta method.

2) Calculate $\boldsymbol{x}^{i+1}$ from Eqs. (7).

3) Repeat steps 1 and 2 until the L_2 norm of the error is less than 10^{-10}.

For large values of λ, the equations are strongly nonlinear, and a good initial guess of t and s becomes crucial. In this case, the solution is obtained by marching in λ from zero to the final value in increments. At the starting point, $\lambda = 0$, the initial guess $\boldsymbol{x}^0$ is set to zero, and the procedure described is implemented until convergence. The limit of $\lambda = 0$ is in fact the creeping flow limit where the flow equation is linear. The value of λ is then incremented, and the current values of $\boldsymbol{x}$ are used as initial guess; then λ is incremented again and so on until the target λ value is reached.

Discussion of Results

This problem has been previously solved by Zitouni and Vatistas[6] using a power series. The latter method of solution is mathematically cumbersome because it involves the evaluation of elliptic integrals, which are given in terms of infinite power series, that must be evaluated numerically. In this Note, we are presenting an alternate solution method that is relatively simpler and provides results that are hardly different with the ones obtained by the former method. Consequently, the conclusions concerning the flow development are similar with those outlined by Zitouni and Vatistas,[6] and only the most salient elements will be briefly given here. For more detailed descriptions of the two flows, see Ref. 6.

The development of the sink and source flows are found to be notably different except for λ close to zero, where both behaved in a manner similar to Poiseuille's flow. For the inflow, as λ increases, the radial velocity is found to flatten in the neighborhood of the midplane progressively expanding toward the walls. In the limiting case where $\lambda \to \infty$, the velocity profile is flat throughout the entire channel. In antithesis to the inflow, the central velocity for the outflow grows with λ, developing a flow reversal near the two disk walls, when $\lambda = \sqrt{(3\pi/2)}$ (Ref. 6).

In Fig. 2, the pressure distributions obtained from the present theory are compared with the observations of Hayes and Tucker[8] and those of Singh[9] for inflow, along with the observations of Moller[10] for an outflow. It is evident from this graph that the two agree reasonably well. This is also apparent for values of $\lambda < \approx 0.5$, and as far as the pressure is concerned, both flows can be assumed to be of Poiseuille's type. Strictly speaking, the coincidence of the two flows happens only when $\lambda \to 0$. For values of λ greater than 0.5, the pressure distributions for the two kinds of flow bifurcate and are seen to be heading into two different directions. As $\lambda \to \infty$, the inflow pressure approaches asymptotically 1.00, while the pressure of the outflow descends monotonically, terminating when a critical value of $\lambda = \sqrt{(3\pi/2)}$ (Ref. 6) is reached, where at this point, pure radial flow ceases to be a reasonable assumption.

In Fig. 3, radial velocity distributions obtained from the present theory are compared with the laser Doppler anemometer

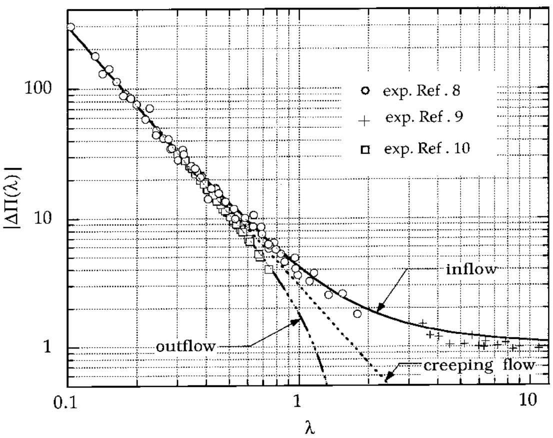

Fig. 2 Pressure variation as a function of λ.

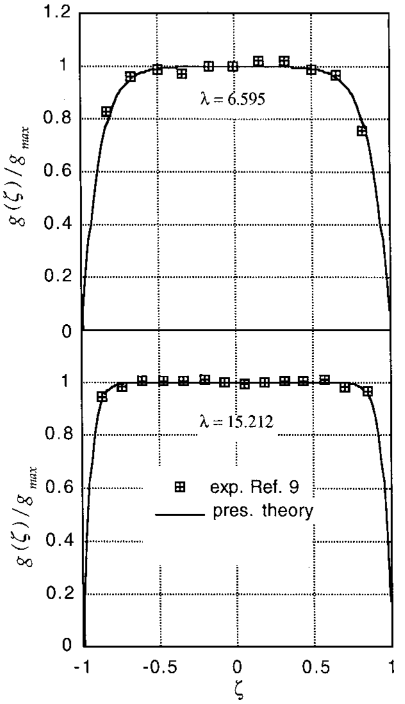

Fig. 3 Comparison of the experimental and theoretical velocity profiles for accelerating flows.

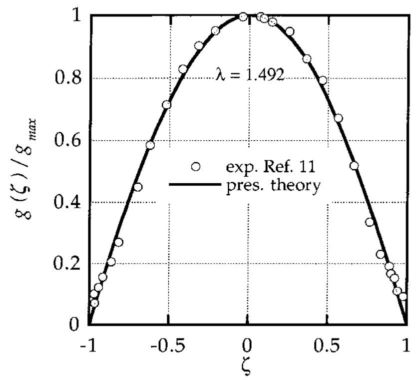

Fig. 4 Comparison of experimental and theoretical velocity profiles for decelerating flows.

measurements of Singh[9] for the case of an accelerating flow, whereas in Fig. 4 the measurements of Tabatabai and Pollard[11] for the decelerating flow are presented. It is clear that, in both cases, the present solution provides a reasonable approximation. It is also fair to say that the same degree of approximation to the accelerating flow can also be achieved via the finite differences, primitive variable, solution of Singh.[9] The latter method, however, involves the axisymmetric case of the Navier–Stokes equations and includes the effects of turbulence via the k–ε model.

Conclusions

The Note reported on numerical solutions of converging and diverging unidirectional flows. The solution to the problem was accomplished using a shooting method, whereby the governing equation was written as a system of three nonlinear first-order ODEs and was solved as an initial value problem via the Runge–Kutta method. The results of the numerical study are shown to compare well with the experimental measurements. The present simple method provided equivalent findings with the previously reported analysis that uses mathematically cumbersome elliptic integrals.

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Examination of Anomalous Shock Velocities in Weakly Ionized Gases

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Introduction

RECENT studies (see Ref. 1 for a review) subtly suggest that new physics may be necessary to explain anomalous drag reduction and shock propagation in a weakly ionized gas (WIG). Several[2–6] studies conclude that shock waves travel faster in the WIG (fractional ionization $\sim 10^{-6}$) than expected from the gas temperature. Other work[7] concludes that anomalous dispersion and attenuation effects cannot be explained only with thermal effects. More recent work (Ref. 8 and references therein) has alternatively argued that temperature gradients and viscous effects in the discharge can explain the shock dispersion, splitting, and attenuation. In this Note, we address the issue of shock speeds and argue that the measured shock speeds are not anomalous. Using standard one-dimensional and inviscid normal shock wave gasdynamics, we use the experimental parameters provided in these studies to show that the observed shock wave speeds are the same as those expected for the given temperatures.

Description

In these experiments, a steady WIG discharge is produced in a localized region of a low-pressure tube, in which the pressures

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